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A DESIGN OF OPTIMUM SCREENING PROCEDURE USING SURROGATE VARIABLE

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In this article, a new profit function based on a surrogate variable of its performance variable is provided to develop an optimum screening procedure for manufacturing. The optimum screening procedure helps producers set up the mean level of manufacturing and the screening limits of a surrogate variable to reach a maximum expected profit per unit. The proposed method is useful when the products in the manufacturing process are classified into different grades and sold in two alternate markets. A cement-packing example is used to illustrate the proposed method, and a numerical study is conducted to evaluate the effects of cost components and distribution parameters on the expected profit per unit. The proposed screening procedure provides a significant improvement over existing methods in term of higher expected profit per unit.

Keywords: Production; screening limits; performance variable; surrogate variable; profit function.

1. Introduction

Automatic production techniques have been successfully applied in today's manufacturing. Products often are inspected before they are shipped to consumers to

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determine whether their quality meets the acceptance level. In most situations, the quality characteristic of a product (or performance variable) is quantitative such as the weight, volume and the geometric dimension of products. If the value of a performance variable is lower than its lower specification limit, it may be sold at a discounted price or reworked. Producers may enhance the mean level in production to decrease the proportion of nonconformities. However, such an adjustment may unavoidably increase the cost. Consequently, the selection of process mean level depends on a tradeoff among the production cost, payoff of conforming products, and the costs incurred owing to nonconforming products.

Many researchers have paid attention on the development of optimum screening procedure. References 24, 3, 8 and 9 considered the problem of optimum process setting for a filling process, in which underfilled or overfilled products are reprocessed at a fixed cost. References 12, 4 and 5 studied sale conditions of products when the measure of performance variable is lower than the specification limit. However, in many situations, it would be impossible or not economical to screen products via performance variable, and producers may then inspect its surrogate variable instead. When a variable that is highly correlated with the performance variable is easier and cheaper to measure, we call this variable the surrogate variable. Many studies in various applications with models developed using surrogate variable can be found in the literature, such as, Refs. 5, 26, 2.

Due to the production variability, manufacturers often sort accepted products into different grades and sell them to different markets. This practice has been used for sporting goods, electronic products, chemical materials and some primary materials such as lumber, wheat, and butter. Products of the same brand name but different grades may be sold at different prices, or marketed in different chain stores or areas. Some economic inspection procedures can be found in Refs. 25, 26, 14, 16, 10 and 6.

Most works in the literature gave detailed discussions for the development of optimum screening procedure using the joint density function of the performance and surrogate variables. However, all these studies considered a constant penalty cost to punish misclassification error of products in a given interval. It is unrealistic to use the same penalty cost for misclassification of product no matter how large an error incurred. In this paper, a new profit function is provided to determine the optimum process mean level and the screening limits of surrogate variable. Assuming that a surrogate variable can be observed and the conditional density function of its performance variable is known, the level or grade of this surrogate variable can be used to construct the new profit function to reach a maximum expected profit per unit. Moreover, the proposed profit function punishes the misclassification error of a product with a penalty cost proportional to the distance between the real quality level of the performance variable and the specification limits. The proposed profit function considers existing methods as special cases and is capable of reaching a larger expected profit per unit.

The rest of this article is organized as follows. In Sec. 2, the optimum screening procedure with a new profit function is discussed. A new profit function containing the selling prices, production cost, inspection cost, rework cost, and penalty costs as components is given. Methods of finding the optimum process mean level and the screening limits of the surrogate variable are provided. An example is presented in Sec. 3 to illustrate the proposed method. Moreover, a numerical study is conducted to evaluate the performance of the proposed method. The effect of cost components and distribution parameters on the expected profit per unit are studied through a sensitivity study. Finally, some concluding remarks are given.

2. Optimum Screening Procedure for a New Profit Model

We consider a process in which products are manufactured and inspected into three quality levels A, B, and C before they are shipped to consumers. Products of grade A are sold to a primary market, products of grade B are sold to a secondary market, and products of grade C are reworked through the same manufacturing process and inspected and classified again. Let L_1 and $L_2(\leq L_1)$ be two specification limits on a performance variable Y for grades A and B. A product with quantity $Y \geq L_1$ and $L_1 > Y \geq L_2$ is sold at fixed prices a_1 and a_2 per unit, respectively, to the primary and secondary markets; and product with $Y < L_2$ is reworked with the same manufacturing process at a rework cost r with $r < a_2 < a_1$. Assume that the production cost per unit is proportional to Y, i.e., when $Y = y$, the cost is $b+cy$, where b is the fixed cost per product and $c(c \geq 0)$ is the unit cost of a quantity of product. Let the performance variable of a reworked product be Y_R. Without loss of generality, we can assume that Y and Y_R are identically and independently normally distributed with mean μ_y and variance σ_y^2, denoted by $N(\mu_y, \sigma_y^2)$.

In the situation when the inspection cost of Y is high, producers would want to run the screening procedure based on its surrogate variable X instead. We now consider the conditional distribution of X, given $Y = y$ as $N(\lambda_1+\lambda_2 y, \sigma^2)$, where λ_1 and λ_2 are known constants. Usually, λ_2 is positive so that X and Y are positively correlated. If X is negatively correlated with Y, one can use a screening variable $-X$ rather than X to develop the optimum screening procedure. Accordingly, the covariate (X, Y) follow a bivariate normal distribution with means (μ_x, μ_y), variances (σ_x^2, σ_y^2) and correlation coefficient $\rho = \sqrt{\lambda_2^2\sigma_y^2/\sigma_x^2}$, where $\mu_x = \lambda_1 + \lambda_2\mu_y$ and $\sigma_x^2 = \lambda_2^2\sigma_y^2 + \sigma^2$.

Let ω_1 and ω_2 be screening limits on X with $\omega_1 \geq \omega_2$. Every accepted product under inspection is classified into one of the three quality grades: (i) if $X \geq \omega_1$, we conclude that $Y \geq L_1$. The product is sold at the price a_1 to the primary market; (ii) if $\omega_2 \leq X < \omega_1$, we conclude that $L_2 \leq Y < L_1$. The product is sold at the price a_2 to the secondary markets; (iii) if $X < \omega_2$, the product is reworked at a rework cost r. Because X is not perfectly correlated with Y, the error of accepting products with $Y < L_1$ or $Y < L_2$ incur penalty costs. A way to discourage a misclassification is to consider the penalty cost proportional to the distance between the real quality level of Y and the specification limits of quality. For example, if a product has the condition $Y < L_1$ but it is misclassified as $Y \geq L_1$, the penalty cost can be taken as $d(L_1 - Y)$, where d is a constant multiplier to the penalty. Alternatively, if a product has the condition $Y < L_2$ but it is misclassified as $L_2 \leq Y < L_1$, the penalty cost can be calculated as $d(L_2 - Y)$. Let $P_G = P_G(Y|x, \mu_y, \omega_1, \omega_2)$ be the profit function per unit, and its expected value is $E(P_G) = E[P_G(Y|x, \mu_y, \omega_1, \omega_2)]$. Accordingly, P_G can be presented by

$$P_G = \begin{cases} a_1 - b - cY - c_x, & Y \geq L_1, \text{ given } X \geq \omega_1, \\ a_1 - b - cY - c_x - d(L_1 - Y), & Y < L_1, \text{ given } X \geq \omega_1, \\ a_2 - b - cY - c_x, & Y \geq L_2, \text{ given } \omega_2 \leq X < \omega_1, \\ a_2 - b - cY - c_x - d(L_2 - Y), & Y < L_2, \text{ given } \omega_2 \leq X < \omega_1, \\ E[P_G(Y_R|x_R, \mu_y, \omega_1, \omega_2)] - r - c_x, & X < \omega_2. \end{cases} \tag{1}$$

Let $d_1 = d(L_1 - Y)$, if $L_2 \leq Y < L_1$, given $X \geq \omega_1$; and let $d_2 = d(L_2 - Y)$, if $Y < L_2$, given $\omega_2 \leq X < \omega_1$, and let $c' = c - d, d'_1 = dL_1$ and $d'_2 = dL_2$, then P_G reduces to P'_G as the following in (2). P'_G is similar to the profit function P_{II} in Lee *et al.*,[15] but P'_G uses the conditional density function to calculate its expected profit and $c' \neq c$.

$$P'_G = \begin{cases} a_1 - b - cY - c_x, & Y \geq L_1, \text{ given } X \geq \omega_1, \\ a_1 - b - c'Y - c_x - d'_1, & Y < L_1, \text{ given } X \geq \omega_1, \\ a_2 - b - cY - c_x, & Y \geq L_2, \text{ given } \omega_2 \leq X < \omega_1, \\ a_2 - b - c'Y - c_x - d'_2, & Y < L_2, \text{ given } \omega_2 \leq X < \omega_1, \\ E[P'_G(Y_R|x_R, \mu_y, \omega_1, \omega_2)] - r - c_x, & X < \omega_2. \end{cases} \tag{2}$$

For parallel comparison, we reduce the proposed profit function of (2) with the condition of $c' = c$. Then P'_G in (2) can be rewritten as

$$P'_G = \begin{cases} a_1 - b - cY - c_x, & Y \geq L_1, \text{ given } X \geq \omega_1, \\ a_1 - b - cY - c_x - d_1, & Y < L_1, \text{ given } X \geq \omega_1, \\ a_2 - b - cY - c_x, & Y \geq L_2, \text{ given } \omega_2 \leq X < \omega_1, \\ a_2 - b - cY - c_x - d_2, & Y < L_2, \text{ given } \omega_2 \leq X < \omega_1, \\ E[P'_G(Y_R|x_R, \mu_y, \omega_1, \omega_2)] - r - c_x, & X < \omega_2. \end{cases} \tag{3}$$

We want to show that the new profit function P'_G performs better than P_{II} with a bigger expected profit per unit even when the underlying distribution is different. In practice, we need to classify the quality of product according to an observed level of surrogate variable. So the conditional density function is used to calculate the expected profit per unit instead of using a joint density function. Let $\delta_i = (\omega_i - \mu_x)/\sigma_x, \Phi_{-i} = \Phi(-\delta_i), \phi_i = \phi(\delta_i)$ and $\Phi_i = \Phi(\delta_i), i = 1, 2$, where ϕ and Φ are the probability density function and cumulative density function of the standard normal distribution, respectively; let $\Psi(\cdot, \cdot|\rho)$ be the standardized bivariate normal distribution with correlation coefficient ρ; let $\Psi_{-1\cdot 1} = \Psi(-\delta_1, \eta|-\rho), \Psi_{-1\cdot 2} = \Psi(-\delta_1, -\eta|\rho)$,

$\Psi_{1\cdot 1} = \Psi(\delta_1, \eta - \chi|\rho), \Psi_{1\cdot 2} = \Psi(\delta_1, \chi - \eta| - \rho)$, $\Psi_{2\cdot 1} = \Psi(\delta_2, \eta - \chi|\rho)$, and $\Psi_{2\cdot 2} = \Psi(\delta_2, \chi - \eta| - \rho)$. Using the proof in Appendix A, we can show that:

(a) $\int_{-\infty}^{\infty} yf(y|X \geq \omega_1)dy = \mu_y + \frac{\rho\sigma_y\phi_1}{\Phi_{-1}}$.
(b) $\int_{-\infty}^{L_1} f(y|X \geq \omega_1)dy = \frac{\Psi_{-1\cdot 1}}{\Phi_{-1}}$.
(c) $\int_{-\infty}^{\infty} yf(y|\omega_2 \leq X < \omega_1)dy = \mu_y + \frac{\rho\sigma_y\phi_{21}}{\Phi_{12}}$.
(d) $\int_{-\infty}^{L_2} f(y|\omega_2 \leq X < \omega_1)dy = \frac{\Psi_{1\cdot 1} - \Psi_{2\cdot 1}}{\Phi_{12}}$.

where $\phi_{21} = \phi_2 - \phi_1$ and $\Phi_{12} = \Phi_1 - \Phi_2$.

It follows that the expected profit of P'_G in (3) can be derived based on the proof in Appendix B as

$$E(P'_G) = \frac{1}{\Phi_{-2}} \left\{ \begin{array}{c} a_1 + a_2 - 2b - 2c_x - c\left(2(L_1 - \eta\sigma_y) + \rho\sigma_y\left[\dfrac{\phi_1}{\Phi_{-1}} + \dfrac{\phi_{21}}{\Phi_{12}}\right]\right) \\ -d_1\dfrac{\Psi_{-1\cdot 1}}{\Phi_{-1}} - d_2\dfrac{\Psi_{1\cdot 1} - \Psi_{2\cdot 1}}{\Phi_{12}} - (r + c_x)\Phi_2 \end{array} \right\}. \tag{4}$$

If $E(P'_G)$ is a unimodal function of η and δ_i, the optimum values of η and δ_i can be determined by solving equations of $\frac{\partial E(P'_G)}{\partial \eta} = 0$ and $\frac{\partial E(P'_G)}{\partial \delta_i} = 0, i = 1, 2,$ simultaneously as the follows:

$$\frac{d_1\Phi_{1.1}\phi(\eta)}{\Phi_{-1}\phi(\eta - \chi)} + \frac{d_2}{\Phi_{12}}\{\Phi_{1\cdot 3} - \Phi_{1\cdot 4}\} - \frac{2c\sigma_y}{\phi(\eta - \chi)} = 0 \tag{5}$$

$$\begin{aligned} c\rho\sigma_y&\left\{\frac{\phi_1 - \delta_1\Phi_{-1}}{\Phi_{-1}^2} + \frac{\delta_1\Phi_{12} - \phi_{21}}{\Phi_{12}^2}\right\} - \frac{d_1}{\Phi_{-1}^2}\{\Phi_{2\cdot 1}\Phi_{-1} - \Psi_{-1\cdot 1}\} \\ &+ \frac{d_2}{\Phi_{12}^2}\{\Phi_{2\cdot 2}\Phi_{12} - \Psi_{1\cdot 1} + \Psi_{2\cdot 1}\} = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} &\frac{c\rho\sigma_y}{\Phi_{12}^2}\{\phi_{21} - \delta_2\Phi_{12}\} - \frac{d_2}{\Phi_{12}^2}\{\Phi_{3\cdot 1}\Phi_{12} - \Psi_{1\cdot 1} + \Psi_{2\cdot 1}\} + (r + c_x) \\ &\quad - \frac{1}{\Phi_{-2}}\left\{a_1 + a_2 - 2b - 2c_x - c\left(2(L_1 - \eta\sigma_y) + \rho\sigma_y\left[\frac{\phi_1}{\Phi_{-1}} + \frac{\phi_{21}}{\Phi_{12}}\right]\right)\right. \\ &\quad \left. - d_1\frac{\Psi_{-1\cdot 1}}{\Phi_{-1}} - d_2\frac{\Psi_{1\cdot 1} - \Psi_{2\cdot 1}}{\Phi_{12}} - (r + c_x)\Phi_2\right\} = 0 \end{aligned} \tag{7}$$

where $\Phi_{1\cdot 1} = \Phi((-\delta_1 + \rho\eta)/\sqrt{1 - \rho^2})$, $\Phi_{1\cdot 3} = \Phi((\delta_1 - \rho(\eta - \chi))/\sqrt{1 - \rho^2})$, $\Phi_{1\cdot 4} = \Phi((\delta_2 - \rho(\eta - \chi))/\sqrt{1 - \rho^2})$, $\Phi_{2\cdot 1} = \Phi((\eta - \rho\delta_1)/\sqrt{1 - \rho^2})$, $\Phi_{2\cdot 2} = \Phi((\eta - \chi - \rho\delta_1)/\sqrt{1 - \rho^2})$, and $\Phi_{3\cdot 1} = \Phi((\eta - \chi - \rho\delta_2)/\sqrt{1 - \rho^2})$. If $E(P'_G)$ is not a unimodal function of η and δ_i, a global searching procedure (see Refs. 21 and 22) is needed to find the optimum values of η and δ_i in a large ranges of η and δ_i.

Because there is no close-form solution for Eqs. (5)–(7), the optimum values of parameters η and δ_i, denoted by $\eta^*_{G'}$ and $\delta^*_{i,G'}, i = 1, 2,$ should be evaluated numerically. A computation approach such as the General-purpose Optimization Method[27] or the L-BFGS-B method[13] is required to obtain these values of $\eta^*_{G'}$ and $\delta^*_{i,G'}, i = 1, 2$. Accordingly, the optimum process mean level of Y based on P'_G, denoted by $\mu^*_{y,G'}$, and the optimal screening limits on X, denoted by $\omega^*_{1,G'}$ and $\omega^*_{2,G'}$, can be determined, respectively by

$$\mu^*_{y,G'} = L_1 - \eta^*_{G'}\sigma_y. \tag{8}$$

$$\omega^*_{i,G'} = \mu_x + \delta^*_{i,G'}\sigma_x, \quad \text{for } i = 1, 2. \tag{9}$$

Using the derivation in Appendix C, we can shown that the expected profit function of P_G in (1) as

$$\begin{aligned} E(P_G) &= E(P'_G; d_1 = d'_1, d'_2 = d'_2) \\ &\quad + \frac{d}{\Phi_{-2}}\left\{\int_{-\infty}^{L_1} yf(y|X \geq \omega_1)dy + \int_{-\infty}^{L_2} yf(y|\omega_2 \leq X \leq \omega_1)dy\right\}, \end{aligned} \tag{10}$$

where $\int_{-\infty}^{L_1} yf(y|X \geq \omega_1)dy$ is a functional of (η, δ_1) and $\int_{-\infty}^{L_2} yf(y|\omega_2 \leq X \leq \omega_1)dy$ is a function of $(\eta, \delta_1, \delta_2)$. Let

$$\begin{aligned} G1 &= \frac{\eta\sigma_y + \mu_y}{\Phi_1}\phi(\eta)(1 - \Phi_{1\cdot 1}), \\ G2 &= \frac{(\eta - \chi)\sigma_y + \mu_y}{\Phi_{12}}\phi(\eta - \chi)(\Phi_{1\cdot 3} - \Phi_{1\cdot 4}), \\ G3 &= \frac{-1}{\Phi_{-1}}\int_{-\infty}^{\eta} \frac{z_2\sigma_y + \mu_y}{2\pi\sqrt{1-\rho^2}} e^{-(\delta_1^2 + z_2^2 + 2\rho\delta_1 z_2)/2(1-\rho^2)}dz_2 \\ &\quad + \frac{\phi_1}{(\Phi_{-1})^2}\int_{\delta_1}^{\infty}\int_{-\infty}^{\eta} \frac{z_2\sigma_y + \mu_y}{2\pi\sqrt{1-\rho^2}} e^{-(z_1^2 + z_2^2 + 2\rho z_1 z_2)/2(1-\rho^2)}dz_2 dz_1, \\ G4 &= \frac{1}{\Phi_{12}}\int_{-\infty}^{\eta-\chi} \frac{z_2\sigma_y + \mu_y}{2\pi\sqrt{1-\rho^2}} e^{-(\delta_1^2 + z_2^2 + 2\rho\delta_1 z_2)/2(1-\rho^2)}dz_2 \\ &\quad + \frac{\phi_1}{(\Phi_{12})^2}\int_{\delta_2}^{\delta_1}\int_{-\infty}^{\eta-\chi} \frac{z_2\sigma_y + \mu_y}{2\pi\sqrt{1-\rho^2}} e^{-(z_1^2 + z_2^2 + 2\rho z_1 z_2)/2(1-\rho^2)}dz_2 dz_1, \\ G5 &= \frac{-1}{\Phi_{12}}\int_{-\infty}^{\eta-\chi} \frac{z_2\sigma_y + \mu_y}{2\pi\sqrt{1-\rho^2}} e^{-(\delta_2^2 + z_2^2 + 2\rho\delta_2 z_2)/2(1-\rho^2)}dz_2 \\ &\quad + \frac{\phi_2}{(\Phi_{12})^2}\int_{\delta_2}^{\delta_1}\int_{-\infty}^{\eta-\chi} \frac{z_2\sigma_y + \mu_y}{2\pi\sqrt{1-\rho^2}} e^{-(z_1^2 + z_2^2 + 2\rho z_1 z_2)/2(1-\rho^2)}dz_2 dz_1, \end{aligned}$$

where $z_1 = (x - \mu_x)/\sigma_x$ and $z_2 = (y - \mu_y)/\sigma_y$. If $E(P_G)$ is a unimodal function of η and δ_i, the optimum values of η and δ_i, denoted by η^*_G and $\delta^*_{i,G} i = 1, 2$, can be determined numerically. Otherwise, a global searching procedure is needed. It can be shown that η^*_G and $\delta^*_{i,G} i = 1, 2$, are the values satisfying the following three equations simultaneously:

$$\frac{\partial}{\partial \eta} E(P'_G; d_1 = d'_1, d_2 = d'_2) + \frac{d}{\Phi_{-2}} \{G1 + G2\} = 0 \tag{11}$$

$$\frac{\partial}{\partial \delta_1} E(P'_G; d_1 = d'_1, d_2 = d'_2) + \frac{d}{\Phi_{-2}} \{G3 + G4\} = 0 \tag{12}$$

$$\frac{\partial}{\partial \delta_2} E(P'_G; d_1 = d'_1, d_2 = d'_2) + \frac{1}{(\Phi_{-2})^2} \left\{ dG5 + \phi_2 \left[\int_{-\infty}^{L_1} y f(y | X \geq \omega_1) dy + \int_{-\infty}^{L_2} y f(y | \omega_2 \leq X \leq \omega_1) dy \right] \right\} = 0 \tag{13}$$

The optimum process mean level of Y based on P_G, denoted by $\mu^*_{y,G}$, and the optimal screening limits on X, denoted by $\omega^*_{1,G}$ and $\omega^*_{2,G}$, can be determined, respectively by

$$\mu^*_{y,G} = L_1 - \eta^*_G \sigma_y, \tag{14}$$

$$\omega^*_{i,G} = \mu_x + \delta^*_{i,G} \sigma_x, \quad \text{for } i = 1, 2. \tag{15}$$

3. Illustration Example and Numerical Results

3.1. *Illustration example*

A cement-packing example is used for illustration. The packing operation of a cement factory consists of two steps with a filling process and an inspection process. Each cement bag is processed by a filling machine, and moved to the loading and dispatching phases on a conveyor belt. Inspection by continuous weighting feeders (CWFs) is performed. A CWF measures the milliampere (mA) of the load cell for each cement bag, denoted by X. Past experience indicates that the measurement of X is positively correlated with the cement bag's weight denoted as the performance variable Y. The current measurement in mA of the load cell is cheaper and easier to measure than measuring the cement bag's weight directly.

For the purpose of performance comparison, this paper adopts cost components and parameters following the suggestions of Lee *et al.*[15] with $\sigma_y^2 = 1.5635$, and the conditional distribution X, given $Y = y$ is $N(4.0 + 0.08y, 0.0125)$. The cost components and the specification limits for Y are $a_1 = 3.25, a_2 = 3.10, r = 0.10, b = 0.10, c = 0.06$, $c_y = 0.04, c_x = 0.004, d_1 = 6.5, d_2 = 6.2, L_1 = 41.5$ and $L_2 = 40$. The corresponding constants are $\sigma_x^2 = 0.0125, \rho = 0.894$ and $\chi = 1.2$. Four profit functions are considered as follows:

Case I: Using the profit function P_I of Lee *et al.*[15] which involves only the performance variable in the profit function.

Case II: Using the profit function P_{II} of Lee *et al.*[15] which involve both the performance and surrogate variables in the profit function but using the joint density function to calculate the expected profit per unit.

Case III: Using the profit function P'_G in (3).

Case IV: Using the profit function P_G in (1).

Table 1 shows the optimum process mean level, the screening limits of surrogate variable and the expected profit per unit for various profit functions in Case I to Case IV. The optimum values of $\eta^*_{II}, \mu^*_{y,II}$ and $E(P_{II})$ in Table 1 are slightly different from the values in Lee *et al.*[15] because the Eq. (8) of Lee *et al.*[15] contains a minor error. Specifically, the notation of "]" is mistakenly placed in front of the component $\phi(\eta - \chi)$. This equation should have been written as

$$\frac{\left[d_1\Phi\left(\frac{-\delta_1+\eta\rho}{(1-\rho^2)^{0.5}}\right)\phi(\eta) + d_2\left\{\Phi\left(\frac{\delta_1+(\chi-\eta)\rho}{(1-\rho^2)^{0.5}}\right) - \Phi\left(\frac{\delta_2+(\chi-\eta)\rho}{(1-\rho^2)^{0.5}}\right)\right\}\phi(\eta-\chi)\right]}{\{c\sigma_y\Phi(-\delta_2)\}} = 1.$$

Table 1 shows that the expected profits per unit, $E(P'_G)$ and $E(P_G)$, are both bigger than those of $E(P_I)$ and $E(P_{II})$.

3.2. *Numerical results*

A numerical study is conducted to compare the expected profits based on the profit functions of P_I, P_{II} of Lee *et al.*[15] and P'_G in (3) when the cost components or parameters of the underlying distribution vary. Table 2 shows that the expected profit $E(P'_G)$ is bigger than those of $E(P_I)$ and $E(P_{II})$. This result supports the claim that the proposed method gains improvement from existing methods.

From Table 2 we observe that $E(P_{II})$ and $E(P'_G)$ increase as χ or ρ increase. $E(P_{II})$ and $E(P'_G)$ decrease as σ increases. When the cost component of b increases,

Table 1. Optimum process mean level, screening limits of surrogate variable and expected profit for various profit functions.

Cases	Parameter estimates	Optimum process mean level & screening limits	Expected profit
Case I	$\eta^*_I = -0.4576$	$\mu^*_{y,I} = 42.0720$ (kg)	$E(P_I) = 0.5272$
Case II	$\eta^*_{II} = -0.4750$	$\mu^*_{y,II} = 42.0938$ (kg)	
	$\delta^*_{1,II} = 0.4680$	$\omega^*_{1,II} = 7.4199$ (mA)	$E(P_{II}) = 0.4613$
	$\delta^*_{2,II} = 0.9023$	$\omega^*_{2,II} = 7.2664$ (mA)	
Case III	$\eta^*_{G'} = -1.4944$	$\mu^*_{y,G'} = 43.3680$ (kg)	
	$\delta^*_{1,G'} = -0.8529$	$\omega^*_{1,G'} = 7.3739$ (mA)	$E(P'_G) = 0.5843$
	$\delta^*_{2,G'} = -1.0966$	$\omega^*_{2,G'} = 7.3466$ (mA)	
Case IV	$\eta^*_G = -1.5370$	$\mu^*_{y,G} = 43.4213$ (kg)	
	$\delta^*_{1,G} = -11.2593$	$\omega^*_{1,G} = 6.2127$ (mA)	$E(P_G) = 0.7916$
	$\delta^*_{2,G} = -0.5278$	$\omega^*_{2,G} = 7.4146$ (mA)	

Table 2. Values of $\mu^*_{y,G'}, E(P_{II})$ and $E(P'_G)$ for various χ, σ_y, ρ, b and c_x.

χ	σ_y	ρ	b	c_x	$\mu^*_{y,G'}$	$E(P_{II})$	$E(P'_G)$
0.7	1.25	0.894	0.1	0.004	43.3862	0.4827	0.5021
0.9	—	—	—	—	43.3810	0.4863	0.5486
1.1	—	—	—	—	43.3704	0.4895	0.5713
1.3	—	—	—	—	43.3619	0.4922	0.5933
1.5	—	—	—	—	43.3035	0.4945	0.6013
1.2	1	0.894	0.1	0.004	42.9797	0.515	0.6238
—	1.2	—	—	—	43.2887	0.4954	0.5909
—	1.4	—	—	—	43.7107	0.4777	0.5684
—	1.6	—	—	—	43.9415	0.4617	0.5487
—	1.8	—	—	—	44.2951	0.4472	0.54
1.2	1.25	0.65	0.1	0.004	44.3802	0.4473	0.8708
—	—	0.725	—	—	44.3786	0.4563	0.8925
—	—	0.8	—	—	44.4085	0.4685	0.893
—	—	0.875	—	—	44.3549	0.4854	0.8983
—	—	0.95	—	—	44.5891	0.5122	0.9457
1.2	1.25	0.894	0.1	0.004	43.3671	0.5029	0.5838
—	—	—	0.2	—	43.5861	0.4029	0.483
—	—	—	0.3	—	43.6814	0.3029	0.3182
—	—	—	0.4	—	43.7787	0.2029	0.2095
—	—	—	0.5	—	44.0110	0.1029	0.1031
1.2	1.25	0.894	0.1	0.002	43.3671	0.505	0.5888
—	—	—	—	0.003	43.3671	0.504	0.5863
—	—	—	—	0.004	43.3671	0.5029	0.5838
—	—	—	—	0.005	43.3631	0.5019	0.5787
—	—	—	—	0.006	43.3608	0.5009	0.5750

Note: "—" indicates the value does not change.

both $E(P_{II})$ and $E(P'_G)$ decrease. Moreover, both $E(P_{II})$ and $E(P'_G)$ are insensitive to the change of the cost component c_x. When the profit function P'_G is used, the optimum process mean level of μ_y decreases as χ increases. The optimum process mean level decreases as σ_y increases. The optimum process mean level decreases first and then increases as the value of b increases. The change of the optimum process mean level is insensitive to the change of c_x.

Numerical results of sensitivity analysis study based on the proposed profit function of P'_G are given in Figs. 1 to 8. Figure 1 shows that the optimum screening limits decrease as χ increases. Figure 2 shows that the width of band between the optimum screening limits decreases as σ_y increases. Figure 3 shows that the width of band between the optimum screening limits decreases first and then increases as b increases. Figure 4 shows that the change of the optimum screening limits are insensitive to the change of c_x. Figure 5 shows that the proportion of primary market decreases and the proportion of secondary market increase as χ increases. But the proportion of reworked products is insensitive to the change of χ. Figure 6 shows that the proportion of primary market decreases and the proportions of secondary

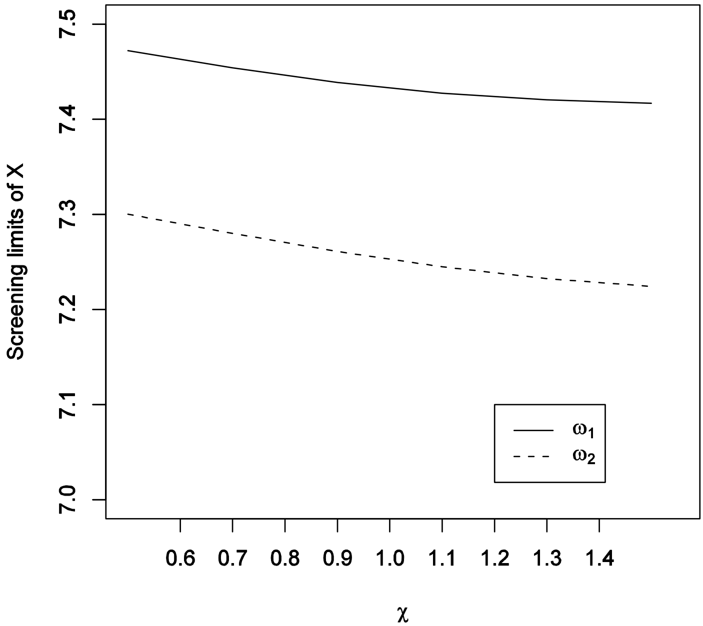

Fig. 1. Effect of χ on ω_1 and ω_2 for P'_G.

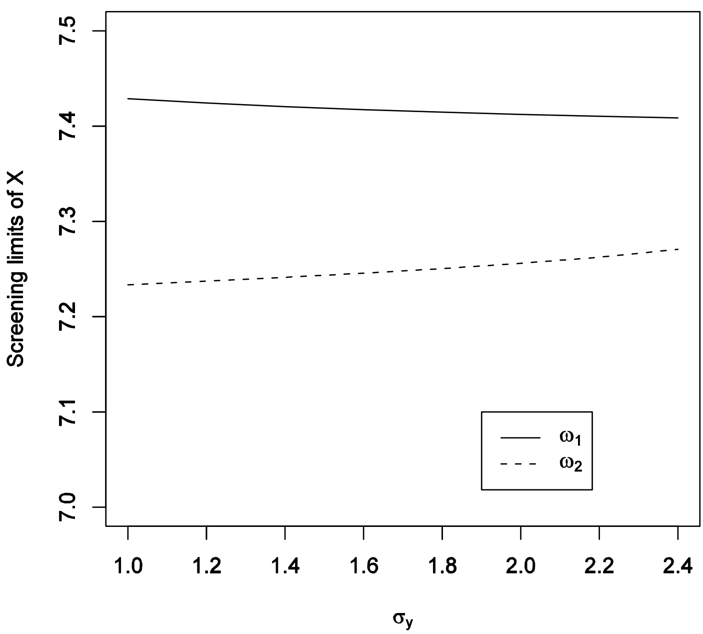

Fig. 2. Effect of σ_y on ω_1 and ω_2 for P'_G.

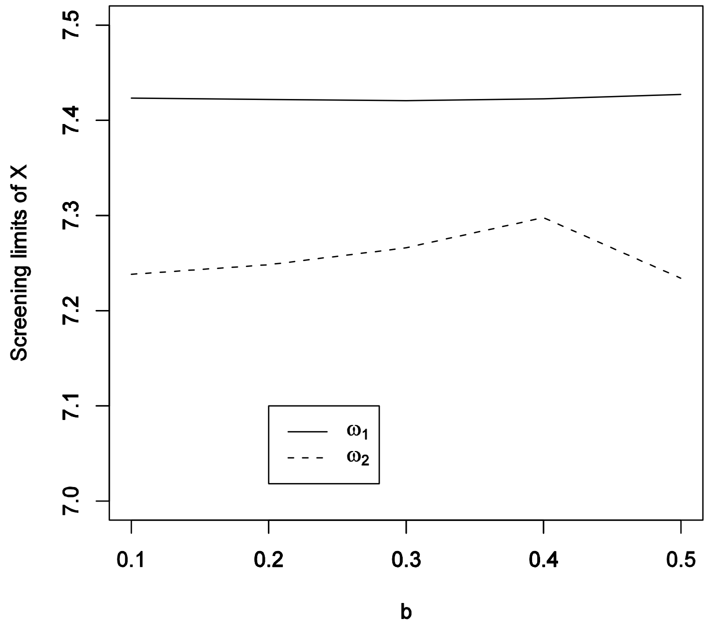

Fig. 3. Effect of b on ω_1 and ω_2 for P'_G.

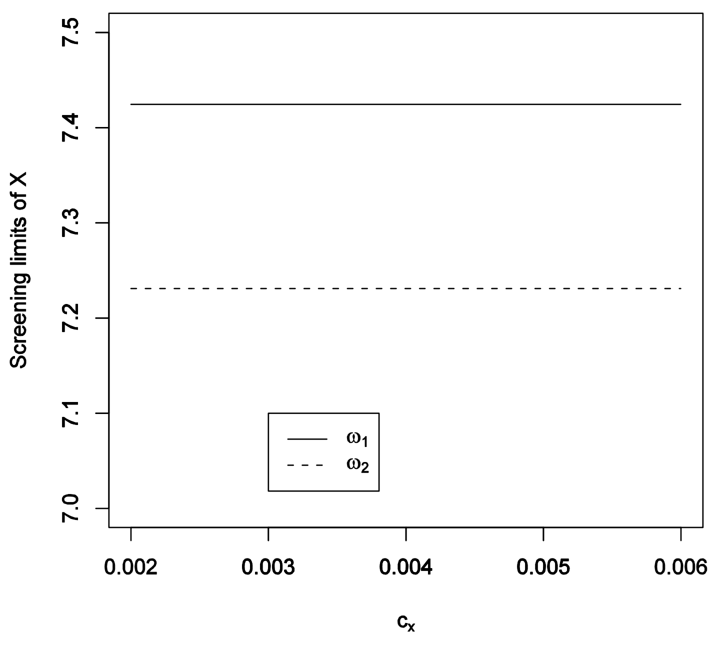

Fig. 4. Effect of c_x on ω_1 and ω_2 for P'_G.

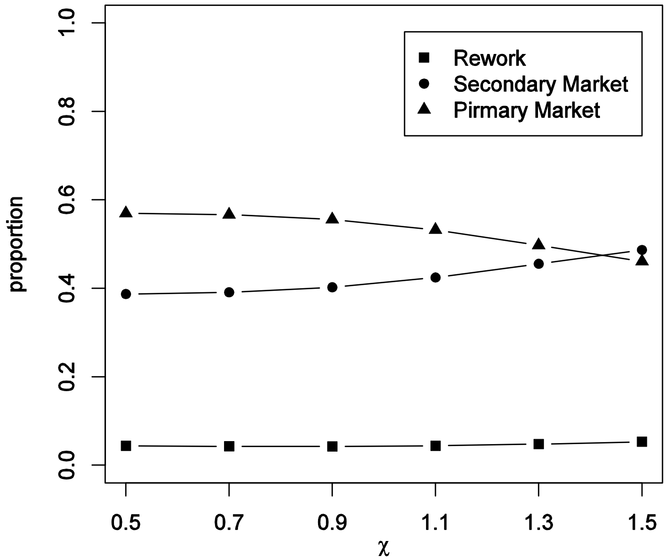

Fig. 5. Effect of χ on the proportions of different markets for P'_G.

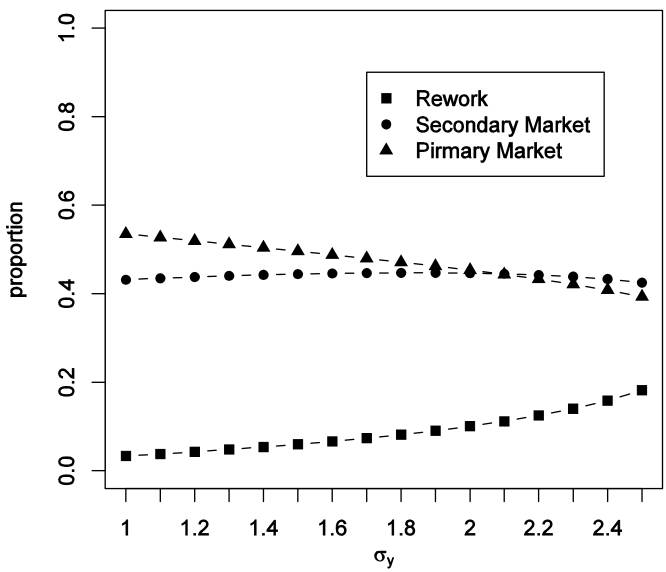

Fig. 6. Effect of σ_y on the proportions of different markets for P'_G.

market and reworked products increase as σ_y increases. Figure 7 shows that the proportion of primary market increases slightly and the proportion of secondary market decreases slightly as ρ increases. But the proportion of reworked products is insensitive to the change of ρ. Figure 8 shows that the proportions of primary market, secondary market and reworked products are insensitive to the change of c_x.

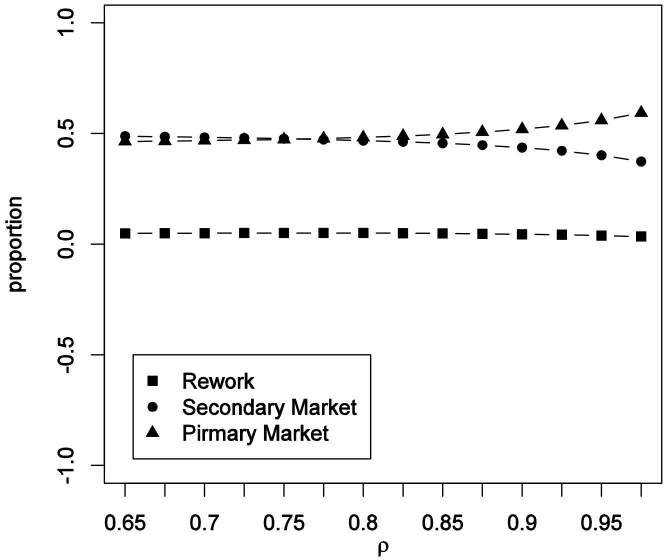

Fig. 7. Effect of ρ on the proportions of different markets for P'_G.

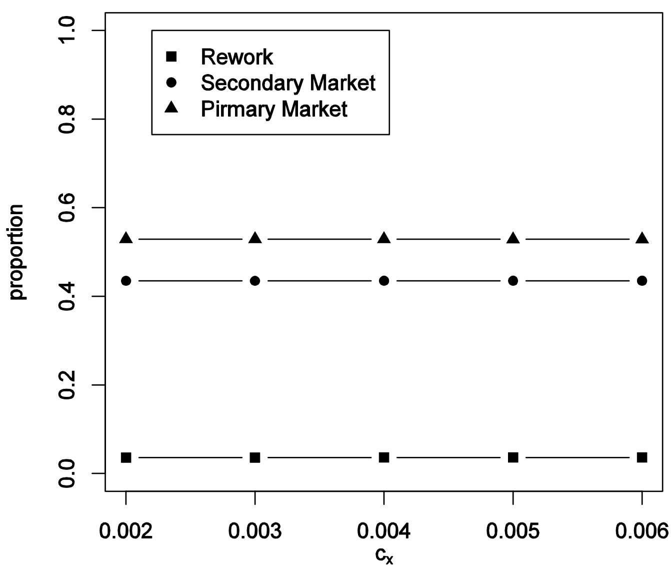

Fig. 8. Effect of c_x on the proportions of different markets for P'_G.

When the profit model P_G is used to develop the optimum screening procedure, we are interested in the effect of the multiplier of penalty cost on the expected profit model. Figure 9 indicates that the optimum screening procedure based on P_G performs the best with a biggest expected profit among all competed optimum

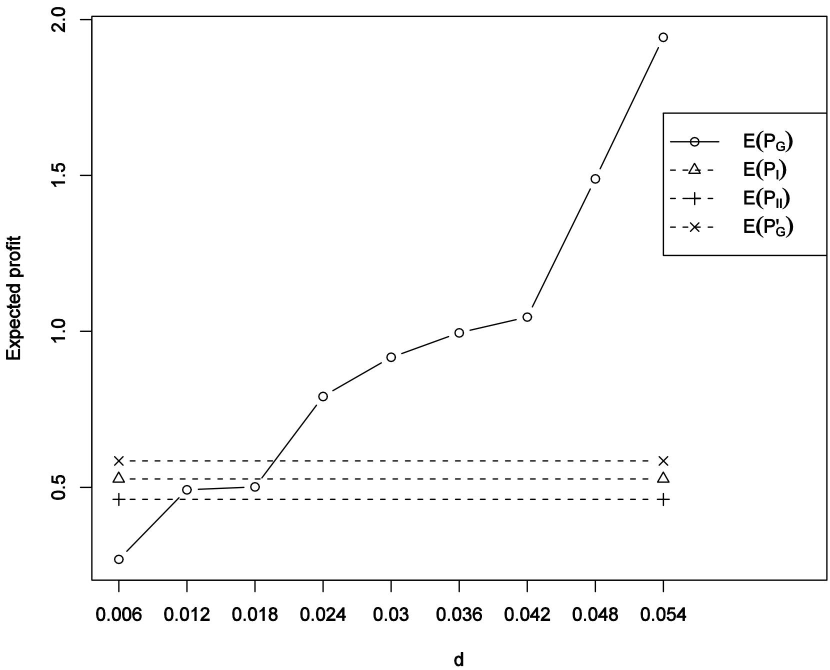

Fig. 9. Compares the expected profits between P_G and other designs.

screening procedures if the multiplier of penalty cost is taken at least 24% of the unit cost of the quantity of a product.

4. Concluding Remarks

We develop an optimum screening procedure to maximize manufacturing profit per unit. A new profit function based on a surrogate variable of its performance variable is introduced to consider selling price, production cost, inspection cost, rework cost and penalty costs incurred due to component misclassifications. A search method is provided to determine the optimum process mean level and the screening limits of the surrogate variable. Some useful formulas in the searching method are derived in the Appendices.

A cement-packing example is used to demonstrate the application of the proposed optimum screening procedure. Through this example, we observe that the proposed method reaches a higher expected profit than the existing methods can. A sensitivity analysis study is conducted and the effects of the cost components and the distribution parameters on the expected profits are discussed. When using the proposed method, users need to have knowledge on the variance and correlation coefficient of the performance variable. Extending the present work to cases where the variance and the correlation coefficient of the performance variable are unknown will be a future topic of studies.

Appendix A: Proof of the Properties with the Profit Function P'_G

(a) It can be shown that the conditional probability distribution of Y, given $X = x$ is normal with mean $\mu_y + \rho\sigma_y/\sigma_y(x-\mu_x)$ and variance $\sigma_y^2(1-p^2)$.

$$
\begin{aligned}
&\int_{-\infty}^{\infty} yf(y|X \geq \omega_1)dy \\
&\quad = \frac{1}{P(X \geq \omega_1)} \int_{\omega_1}^{\infty} \int_{-\infty}^{\infty} yf(x,y)dydx \\
&\quad = \frac{1}{\Phi_{-1}} \int_{\omega_1}^{\infty} \int_{-\infty}^{\infty} yf(y|x)dyf(x)dx \\
&\quad = \frac{1}{\Phi_{-1}} \int_{\omega_1}^{\infty} \left[\mu_y + \rho\frac{\sigma_y}{\sigma_x}(x-\mu_x)\right] f(x)dx \\
&\quad = \frac{1}{\Phi_{-1}} \left\{ \left(\mu_y - \rho\frac{\sigma_y}{\sigma_x}\mu_x\right) \int_{\omega_1}^{\infty} f(x)dx + \rho\frac{\sigma_y}{\sigma_x} \int_{\omega_1}^{\infty} xf(x)dx \right\}. \qquad \text{(A.1)}
\end{aligned}
$$

Let $z = (x-\mu_x)/\sigma_x$ and $x = \mu_x + z\sigma_x$. Equation (A1) can be rewritten as

$$
\begin{aligned}
&\mu_y - \rho\frac{\sigma_y}{\sigma_x}\mu_x + \rho\frac{\sigma_y}{\sigma_x\Phi_{-1}} \int_{\delta_1}^{\infty} (\mu_x + z\sigma_x)\phi(z)\sigma_x dz \\
&\quad = \mu_y - \rho\frac{\sigma_y}{\sigma_x}\mu_x + \left\{ \rho\frac{\sigma_y}{\sigma_x\Phi_{-1}} \int_{\delta_1}^{\infty} (\mu_x + z\sigma_x)\frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz \right\} \\
&\quad = \mu_y - \rho\sigma_y\frac{\phi_1}{\Phi_{-1}}
\end{aligned}
$$

(b)
$$\int_{-\infty}^{L_1} f(y|X \geq \omega_1)dy = \frac{P(X \geq \omega_1, Y < L_1)}{P(X \geq \omega_1)} = \frac{\Psi_{-1\cdot 1}}{\Phi_{-1}}.$$

(c)

$$
\begin{aligned}
\int_{-\infty}^{\infty} yf(y|\omega_2 \leq X < \omega_1)dy &= \frac{1}{\Phi_{12}} \int_{\omega_2}^{\omega_1} \int_{-\infty}^{\infty} yf(y|x)dyf(x)dx \\
&= \frac{1}{\Phi_{12}} \int_{\omega_2}^{\omega_1} \left[\mu_y + \rho\frac{\sigma_y}{\sigma_x}(x-\mu_x)\right] f(x)dx \\
&= \mu_y - \rho\frac{\sigma_y}{\sigma_x}\mu_x + \rho\frac{\sigma_y}{\sigma_x\Phi_{12}} \int_{\omega_2}^{\omega_1} xf(x)dx \qquad \text{(A.2)}
\end{aligned}
$$

Equation (A2) can be rewritten as

$$\mu_y - \rho\frac{\sigma_y}{\sigma_x}\mu_x + \left\{ \rho\frac{\sigma_y}{\sigma_x\Phi_{12}} \int_{\delta_2}^{\delta_1} (\mu_x + z\sigma_x)\frac{1}{\sqrt{2\pi}}e^{-z^2/2}dz \right\} = \mu_y - \rho\sigma_y\frac{\phi_{21}}{\Phi_{12}}$$

(d)
$$\int_{-\infty}^{L_2} f(y|\omega_2 \leq X < \omega_1)dy = \frac{P(\omega_2 \leq X < \omega_1, Y < L_2)}{P(\omega_2 \leq X < \omega_1)} = \frac{\Psi_{1\cdot 1} - \Psi_{2\cdot 1}}{\Phi_{12}}.$$

Appendix B: Proof of $E(P'_G)$

The expected profit per product is given by

$$\begin{aligned}E(P'_G) = &\int_{L_1}^{\infty}(a_1-b-cy-c_x)f(y|x\geq\omega_1)dy\\&+\int_{-\infty}^{L_1}(a_1-b-cy-c_x-d_1)f(y|x\geq\omega_1)dy\\&+\int_{L_2}^{\infty}(a_2-b-cy-c_x)f(y|\omega_2\leq x<\omega_1)dy\\&+\int_{-\infty}^{L_2}(a_2-b-cy-c_x-d_2)f(y|\omega_2\leq x<\omega_1)dy\\&+\{E(P'_G)-r-c_x\}p(x<\omega_2).\end{aligned}$$

Accordingly,

$$\begin{aligned}E(P'_G)\Phi_{-2} = (a_1-b-c_x)&\int_{-\infty}^{\infty}f(y|\geq\omega_1)dy\\&+(a_2-b-c_x)\int_{-\infty}^{\infty}f(y|\omega_2\leq x<\omega_1)dy-d_1\int_{-\infty}^{L_1}f(y|x\geq\omega_1)dy\\&-c\left\{\int_{-\infty}^{\infty}yf(y|x\geq\omega_1)dy+\int_{-\infty}^{\infty}yf(y|\omega_2\leq x<\omega_1)dy\right\}\\&-d_2\int_{-\infty}^{L_2}f(y|\omega_2\leq x<\omega_1)dy-(r+c_x)\Phi_2.\end{aligned}$$

Accordingly, we obtain

$$\begin{aligned}E(P'_G) = \frac{1}{\Phi_{-2}}&\left\{a_1+a_2-2b-2c_x-c\left(2(L_1-\eta\sigma_y)+\rho\sigma_y\left[\frac{\phi_1}{\Phi_{-1}}+\frac{\phi_{21}}{\Phi_{12}}\right]\right)\right.\\&\left.-d_1\frac{\Psi_{-1\cdot1}}{\Phi_{-1}}-d_2\frac{\Psi_{1\cdot1}-\Psi_{2\cdot1}}{\Phi_{12}}-(r+c_x)\Phi_2\right\}.\end{aligned}$$

Appendix C. Proof of $E(P_G)$

The expected profit per product is given by

$$\begin{aligned}E(P_G) = &\int_{L_1}^{\infty}(a_1-b-cy-c_x)f(y|x\geq\omega_1)dy\\&+\int_{-\infty}^{L_1}(a_1-b-c'y-c_x-d'_1)f(y|x\geq\omega_1)dy\\&+\int_{L_2}^{\infty}(a_2-b-cy-c_x)f(y|\omega_2\leq x<\omega_1)dy\end{aligned}$$

$$+ \int_{-\infty}^{L_2} (a_2 - b - c'y - c_x - d_2') f(y|\omega_2 \le x < \omega_1) dy$$
$$+ \{E(P_G) - r - c_x\} p(x < \omega_2).$$

Accordingly,

$$\begin{aligned}
E(P_G)\Phi_{-2} = {} & (a_1 - b - c_x) \int_{-\infty}^{\infty} f(y| \ge \omega_1) dy \\
& + (a_2 - b - c_x) \int_{-\infty}^{\infty} f(y|\omega_2 \le x < \omega_1) dy - d_1' \int_{-\infty}^{L_1} f(y|x \ge \omega_1) dy \\
& - c\left\{ \int_{-\infty}^{\infty} y f(y|x \ge \omega_1) dy + \int_{-\infty}^{\infty} y f(y|\omega_2 \le x < \omega_1) dy \right\} \\
& - d_2' \int_{-\infty}^{L_2} f(y|\omega_2 \le x < \omega_1) dy - (r + c_x)\Phi_2 \\
& + d\left\{ \int_{-\infty}^{L_1} y f(y|x \ge \omega_1) dy + \int_{-\infty}^{L_2} y f(y|\omega_2 \le x < \omega_1) dy \right\}.
\end{aligned}$$

We obtain

$$\begin{aligned}
E(P_G) = {} & E(P_G'; d_1 = d_1', d_2 = d_2') \\
& + \frac{d}{\Phi_{-2}} \left\{ \int_{-\infty}^{L_1} y f(y \,|\, X \ge \omega_1) dy + \int_{-\infty}^{L_2} y f(y|\omega_2 \le X < \omega_1) dy \right\},
\end{aligned}$$

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